

Representing data by sparse combination of contextual data points for classification

Jingyan Wang^{1,2,3}, Yihua Zhou⁴, Ming Yin⁵, Shaochang Chen⁵, and Benjamin Edwards⁶

¹ National Time Service Center, Chinese Academy of Sciences, Xi' an, Shaanxi 710600 , China

² Graduate University of Chinese Academy of Sciences, Beijing 100049, China

³ Provincial Key Laboratory for Computer Information Processing Technology, Soochow University Suzhou 215006, China
jingbinwang1@outlook.com

⁴ Department of mechanical engineering and mechanics, Lehigh University, Bethlehem, PA 18015, USA

⁵ Electronic Engineering College, Naval University of Engineering, Wuhan 430033, China

⁶ Department of Computer Science, Sam Houston State University, Huntsville, TX 77341, USA
benjamin.edwards1@hotmail.com

Abstract. In this paper, we study the problem of using contextual data points of a data point for its classification problem. We propose to represent a data point as the sparse linear reconstruction of its context, and learn the sparse context to gather with a linear classifier in a supervised way to increase its discriminative ability. We proposed a novel formulation for context learning, by modeling the learning of context reconstruction coefficients and classifier in a unified objective. In this objective, the reconstruction error is minimized and the coefficient sparsity is encouraged. Moreover, the hinge loss of the classifier is minimized and the complexity of the classifier is reduced. This objective is optimized by an alternative strategy in an iterative algorithm. Experiments on three benchmark data set show its advantage over state-of-the-art context-based data representation and classification methods.

Key words: Pattern classification, Context learning, Nearest neighbors, and Sparse regularization

1 Introduction

Pattern classification is a major problem in machine learning research [32, 5, 6, 13]. The two most important topics of pattern classification are data representation and classifier learning. Zhang et al. proposed an efficient multi-model classifier for large scale Bio-sequence localization prediction [36]. Zhang et al. developed and optimized association rule mining algorithms and implemented

them on paralleled micro-architectural platforms [39, 38]. Most data representation and classification methods are based on single data point. When one data point is considered for representation and classification, all other data points are ignored. However, the other data points other than the data point under consideration, which are called contextual data points, may play important roles in its representation and classification. It is necessary to explore the contexts of data points when they are represented and/or classified. In this paper, we investigate the problem of learning effective representation of a data point from its context guided by its class label, and proposed a novel supervised context learning method using sparse regularization and linear classifier learning formulation.

We propose a novel method to explore the context of a data point, and use it to represent it. We use its k nearest neighbors as its context, and try to reconstruct it by the data points in its context. The reconstruction errors are imposed to be spares. Moreover, the reconstruction result is used as the new representation of this data point. We apply a linear function to predict its class label from the sparse reconstruction of its context. The motivation of this contribution is that for each data point, only a few data points in its context is of the same class as itself. To find the critical contextual data points, we proposed to learn the classifier together with the sparse context. We model this problem as a minimization problem. In this problem, the context reconstruction error, reconstruction sparsity, classification error, and classifier complexity are minimized simultaneously. We also propose a novel iterative algorithm to solve this minimization problem. We first reformulate it as the Lagrange formula, and then use an alternative optimization method to solve it.

This paper is organized as follows. In section 2, we introduce the proposed method. In section 3, we evaluate the proposed method experimentally. In section 4, this paper is concluded with future works.

2 Proposed method

We consider a binary classification problem, and a training set of n data points are given as $\{(\mathbf{x}_i, y_i)\}_{i=1}^n$, where $\mathbf{x}_i \in \mathbb{R}^d$ is a d -dimensional feature vector of the i -th data point, and $y_i \in \{+1, -1\}$ is the class label of the i -th point. To learn from the context of the i -th data point, we find its k nearest neighbors and denote them as $\{\mathbf{x}_{ij}\}_{j=1}^k$, where \mathbf{x}_{ij} is the j -th nearest neighbor of the i -th point. They are further organized as a $d \times k$ matrix $X_i = [\mathbf{x}_{i1}, \dots, \mathbf{x}_{ik}] \in \mathbb{R}^{d \times k}$, where the j -th column is \mathbf{x}_{ij} . We represent \mathbf{x}_i by linearly reconstructing it from its contextual points as

$$\mathbf{x}_i \approx \hat{\mathbf{x}}_i = \sum_{j=1}^k \mathbf{x}_{ij} v_{ij} = X_i \mathbf{v}_i \quad (1)$$

where $\hat{\mathbf{x}}_i$ is its reconstruction, and v_{ij} is the reconstruction coefficient of the j -th nearest neighbor. $\mathbf{v}_i = [v_{i1}, \dots, v_{ik}]^\top \in \mathbb{R}^k$ is the reconstruction coefficient vector of the i -th data point. The reconstruction coefficient vectors of all the training points are organized in reconstruction coefficient matrix $V = [\mathbf{v}_1, \dots, \mathbf{v}_n] \in \mathbb{R}^{k \times n}$.

$\mathbb{R}^{k \times n}$, with its i -th column as \mathbf{v}_i . To solve the reconstruction coefficient vectors, we propose the following minimization problem,

$$\min_V \left\{ \beta \sum_{i=1}^n \|\mathbf{x}_i - X_i \mathbf{v}_i\|_2^2 + \gamma \sum_{i=1}^n \|\mathbf{v}_i\|_1 \right\}, \quad (2)$$

where β and γ are trade-off parameters. In the objective of this problem, the first term is to minimize the reconstruction error measured by a squared ℓ_2 norm penalty between \mathbf{x}_i and $X_i \mathbf{v}_i$, and the second term is a ℓ_1 norm penalty to the contextual reconstruction coefficient vector \mathbf{v}_i .

We design a classifier to classify the i -th data point,

$$f(\hat{\mathbf{x}}_i) = \mathbf{w}^\top \hat{\mathbf{x}}_i = \mathbf{w}^\top X_i \mathbf{v}_i \quad (3)$$

where $\mathbf{w} \in \mathbb{R}^d$ is the classifier parameter vector. The following optimization problem is proposed to learn \mathbf{w} ,

$$\begin{aligned} \min_{\mathbf{w}, V, \boldsymbol{\xi}} \quad & \left\{ \frac{1}{2} \|\mathbf{w}\|_2^2 + \alpha \sum_{i=1}^n \xi_i \right\} \\ \text{s.t.} \quad & 1 - y_i (\mathbf{w}^\top X_i \mathbf{v}) \leq \xi_i, \xi_i \geq 0, i = 1, \dots, n, \end{aligned} \quad (4)$$

where $\frac{1}{2} \|\mathbf{w}\|_2^2$ is the squared ℓ_2 norm regularization term to reduce the complexity of the classifier, ξ_i is the slack variable for the hinge loss of the i -th training point, $\boldsymbol{\xi} = [\xi_1, \dots, \xi_n]^\top$ and α is a tradeoff parameter.

The overall optimization problem is obtained by combining the problems in both (2) and (4) as

$$\begin{aligned} \min_{\mathbf{w}, V, \boldsymbol{\xi}} \quad & \left\{ \frac{1}{2} \|\mathbf{w}\|_2^2 + \alpha \sum_{i=1}^n \xi_i + \beta \sum_{i=1}^n \|\mathbf{x}_i - X_i \mathbf{v}_i\|_2^2 + \gamma \sum_{i=1}^n \|\mathbf{v}_i\|_1 \right\} \\ \text{s.t.} \quad & 1 - y_i (\mathbf{w}^\top X_i \mathbf{v}) \leq \xi_i, \xi_i \geq 0, i = 1, \dots, n. \end{aligned} \quad (5)$$

According to the dual theory of optimization, the following dual optimization problem is obtained,

$$\begin{aligned} \max_{\boldsymbol{\delta}, \boldsymbol{\epsilon}} \min_{\mathbf{w}, V, \boldsymbol{\xi}} \quad & \left\{ \frac{1}{2} \|\mathbf{w}\|_2^2 + \alpha \sum_{i=1}^n \xi_i + \beta \sum_{i=1}^n \|\mathbf{x}_i - X_i \mathbf{v}_i\|_2^2 + \gamma \sum_{i=1}^n \|\mathbf{v}_i\|_1 \right. \\ & \left. + \sum_{i=1}^n \delta_i (1 - y_i (\mathbf{w}^\top X_i \mathbf{v}_i) - \xi_i) - \sum_{i=1}^n \epsilon_i \xi_i \right\}, \\ \text{s.t.} \quad & \boldsymbol{\delta} \geq 0, \boldsymbol{\epsilon} \geq 0, \end{aligned} \quad (6)$$

where $\boldsymbol{\delta} = [\delta_1, \dots, \delta_n]^\top$, and $\boldsymbol{\epsilon} = [\epsilon_1, \dots, \epsilon_n]^\top$ are Lagrange multipliers. By setting the partial derivative of \mathcal{L} with regard to \mathbf{w} and ξ_i to zeros, we have

$$\begin{aligned}
\mathbf{w} &= \sum_{i=1}^n \delta_i y_i X_i \mathbf{v}_i. \\
\alpha - \delta_i &= \epsilon_i \\
\Rightarrow \alpha &\geq \delta_i.
\end{aligned} \tag{7}$$

We substitute (7) to (6) to eliminate \mathbf{w} and δ ,

$$\begin{aligned}
\max_{\delta} \min_V \left\{ -\frac{1}{2} \sum_{i,j=1}^n \delta_i \delta_j y_i y_j \mathbf{v}_i^\top X_i^\top X_j \mathbf{v}_j + \beta \sum_{i=1}^n \|\mathbf{x}_i - X_i \mathbf{v}_i\|_2^2 \right. \\
\left. + \gamma \sum_{i=1}^n \|\mathbf{v}_i\|_1 + \sum_{i=1}^n \delta_i \right\} \\
s.t. \ \alpha \geq \delta \geq 0.
\end{aligned} \tag{8}$$

where $\alpha = [\alpha, \dots, \alpha]^\top$ is a n dimensional vector of all α elements. We solve this problem with the alternate optimization strategy. In each iteration of an iterative algorithm, we fix δ first to solve V , and then fix V to solve δ .

Solving V When δ is fixed and only V is considered, we solve $\mathbf{v}_i|_{i=1}^n$ one by one, (8) is further reduced to

$$\min_{\mathbf{v}_i} \left\{ -\frac{1}{2} \sum_{i,j=1}^n \delta_i \delta_j y_i y_j \mathbf{v}_i^\top X_i^\top X_j \mathbf{v}_j + \beta \|\mathbf{x}_i - X_i \mathbf{v}_i\|_2^2 + \gamma \|\mathbf{v}_i\|_1 \right\}. \tag{9}$$

This problem could be solved efficiently by the modified feature-sign search algorithm proposed by Gao et al. [2].

Solving δ When V is fixed and only δ is considered, the problem in (8) is reduced to

$$\begin{aligned}
\max_{\delta} \left\{ -\frac{1}{2} \sum_{i,j=1}^n \delta_i \delta_j y_i y_j \mathbf{v}_i^\top X_i^\top X_j \mathbf{v}_j + \sum_{i=1}^n \delta_i \right\} \\
s.t. \ \alpha \geq \delta \geq 0.
\end{aligned} \tag{10}$$

This problem is a typical constrained quadratic programming (QP) problem, and it can be solved efficiently by the active set algorithm.

3 Experiments

In this section, we evaluate the proposed supervised sparse context learning (SSCL) algorithm on several benchmark data sets.

3.1 Experiment setup

In the experiments, we used three data sets, which are introduced as follows:

- **MANET loss data set:** The packet losses of the receiver in mobile Ad hoc networks (MANET) can be classified into three types, which are wireless random errors caused losses, the route change losses induced by node mobility and network congestion. We collect 381 data points for the congestion loss, 458 for the route change loss, and 516 data points for the wireless error loss for this data set. Thus in the data set, there are 1355 data points in total. To extract the feature vector each data point, we calculate 12 features from each data point as in [1], and concatenate them to form a vector.
- **Twitter data set:** The second data set is a Twitter data set. The target of this data set is to predict the gender of the twitter user, male or female, given one of his/her Twitter message. We collected 53,971 twitter messages in total, and among them there are 28,012 messages sent by male users, and 25,959 messages sent by female users. To extract features from each Twitter message, we extract Term features, linguistic features, and medium diversity features as gender-specific features as in [8].
- **Arrhythmia data set:** The third data set is publicly available at <http://archive.ics.uci.edu/ml/datasets/Arrhythmia>. In this data set, there are 452 data points, and they belongs to 16 different classes. Each data point has a feature vector of 279 features.

To conduct the experiments, we used the 10-fold cross validation.

3.2 Experimental Results

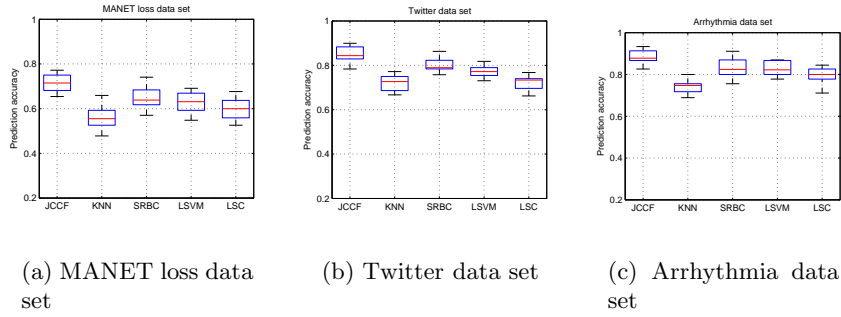


Fig. 1. Boxplots of prediction accuracy of different context-based algorithms.

Since the proposed algorithm is a context-based classification and sparse representation method, we compared the proposed algorithm to three popular

context-based classifiers, and one context-based sparse representation method. The three context-based classifiers are traditional k -nearest neighbor classifier (KNN), sparse representation based classification (SRBC) [26], and Laplacian support vector machine (LSVM) [11]. The context-based sparse representation method is Gao et al.'s Laplacian sparse coding (LSC) [3]. The boxplots of the 10-fold cross validation of the compared algorithms are given in figure 1. From the figures, we can see that the proposed method SSCL outperforms all the other methods on all three data sets. The second best method is SRBC, which also uses sparse context to represent the data point. This is a strong evidence that learning a supervised sparse context is critical for classification problem.

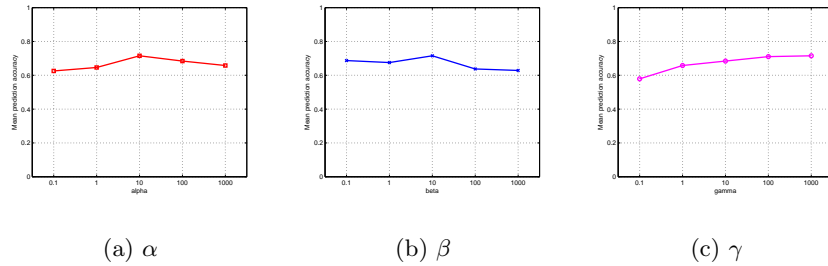


Fig. 2. Parameter sensitivity curves.

Sensitivity to parameters In the proposed formulation, there are three trade-off parameters, α , β , and γ . We plot the curve of mean prediction accuracies against different values of parameters, and show them in figure 2. From figure 2(a) and 2(b), we can see the accuracy is stable to the parameter α and β . From figure 2(c), we can see a larger γ leads to better classification performances.

4 Conclusion and future works

In this paper, we study the problem of using context to represent and classify data points. We propose to use a sparse linear combination of the data points in the context of a data point to represent itself. Moreover, to increase the discriminative ability of the new representation, we develop an supervised method to learn the sparse context by learning it and a classifier together in an unified optimization framework. Experiments on three benchmark data sets show its advantage over state-of-the-art context-based data representation and classification methods. In the future, we will extend the proposed method to applications of information security [33, 27, 30, 29, 28, 31, 34], bioinformatics [25, 24, 23, 12, 15, 14, 7, 37, 7], computer vision [16, 17], and big data analysis using high performance computing [43, 18, 9, 35, 4, 41, 40, 39, 38, 35, 10, 42, 21, 20, 43, 19, 22].

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